Phil 2310 Assignment 10 plus
Fall 2010

Problems from this extra credit homework assignment are due by Wed, Dec 8th – one week after hm 10 is due and one week before the final. You may give it to me by email or in person (come to my office 228 GWS) or turn it in to the philosophy department office in 218 GWS during business hours. You may do any number of these problems to receive credit - you do not have to do all of them.

If you received a score of 70 or under on the second exam, you may get credit for any of these problems. If you received a score of between 71 and 89 (inclusive) on the second exam, you may get credit for any problem marked with a '*' or '**'. If you received a 90 or above on the second exam, you may get extra credit only for those problems marked '**'.

Not doing these problems will not hurt your grade. By this I mean if any curve is applied to the grades at all, it would be calculated before any extra credit is taken into account. I would estimate that doing all of these problems might be worth as much as half of required homework assignment. They will be good practice for the final in any case.

Part I:

*1) Do 11.40 in the textbook

When I talked about quantifier equivalences, (Lecture 22 Wed, Oct 20th) I mentioned that the hardest ones to understand were existentially quantified conditionals. Here is a fact:

$$\exists x\, P(x) \to \exists x\, Q(x) \text{ implies } \exists x(P(x) \to Q(x)) \text{ but not in the other direction.}$$

2) Prove this by giving a formal proof from left to right and give a counterexample interpretation from right to left. You may use FO Con for DeMQ.

**3) Prove the above by giving a formal proof from left to right and give a counterexample interpretation from right to left. But you may use only the basic rules of $\mathcal{F}$ (plus Taut Con).

More proofs where DeMQ helps.
*Prove the following sequents. You may use FO Con for DeMQ.
**Prove them without FO Con.

4) $\forall x[\exists y\, R(x,y) \to \exists y\, R(y,x)],\ \exists x\forall y\, \neg R(y,x) \vdash \exists x\forall y\, \neg R(x,y)$
5) $\forall x\exists y(R(x,y) \land S(x,y)),\ \forall x(\exists y\, S(y,x) \to P(x)),\ \forall x[(P(x) \land \exists y\, R(x,y)) \to Q(x)]$
$\vdash \forall x\exists y[Q(y) \land R(x,y)]$
6) $\forall x(\exists y\, R(x,y) \to \exists y\, S(x,y)) \vdash \forall x\forall y\exists z[R(x,y) \to S(x,z)]$

Prove the following theorem:

**Prove it without FO con

7) $\vdash \neg\forall x[P(x) \rightarrow \exists y(Q(y) \wedge R(x,y))] \leftrightarrow \exists x[P(x) \wedge \forall y(Q(y) \rightarrow \neg R(x,y))]$

Find the handout labeled “Theorems that sound really funny”.

8) Translate sentences 1-13 on this handout into FOL
9) Prove that these sentences are theorems. You may use FO Con for DeMQ or NI.
**10) Prove facts 6(a+b). You may use FO Con for DeMQ or NI.
**11) Prove that the facts corresponding to sentences 1,5,7(a), 9, 10, 12, 13 hold. No FO Con. For 12, give a model to show consistency.

Part II: More on Axioms for Genealogy (some stuff repeated from hm 10 but you need to refer to it here)

Here some meaning postulates for genealogical relationships where P(x,y) is supposed to capture that x is a parent of y:

S: $\forall x\forall y(S(x,y)\leftrightarrow\exists z(P(z,x) \wedge P(z,y) \wedge x\neq y))$ *sibling*
G: $\forall x\forall y(G(x,y)\leftrightarrow\exists z(P(x,z) \wedge P(z,y)))$ *grandparent*
U: $\forall x\forall y(U(x,y)\leftrightarrow\exists z(P(z,y) \wedge S(z,x)))$ *uncle/aunt*
C: $\forall x\forall y(C(x,y)\leftrightarrow\exists z(P(z,y) \wedge U(z,x)))$ *first cousin*

You can prove a good number of things from these postulates. Prove that each of the following follows from S-C. To do this, start from no premises, but anytime you feel like it, you may write any of S-C and simply cite “S” or “G” or whatever is appropriate.

1) If Adam has a cousin, then one of his parents has a sibling.
$\exists x\, C(x,a) \rightarrow \exists x(P(x,a) \wedge \exists y\, S(y,x))$
2) First cousins share a grandparent. $\forall x\forall y(C(x,y) \rightarrow \exists z(G(z,x) \wedge G(z,y)))$

3) Using the above predicates, formalize the claim that for any two of Bettie’s grandchildren, these two must be either siblings or cousins.

4) Prove claim #3

These postulates never allow you to prove any ‘negative’ claims (except that ‘sibling’ is irreflexive). For example, each of the following is consistent with S-C: Adam is his own uncle, Bob’s uncle is the parent of Bob’s grandfather, Christine has a sibling who is also her cousin and also her grandparent.

You might think that these should be logically ruled out. But each of those is consistent if you have the right kind of incestuous relationships. On the other hand, barring time travel, the meaning of parent <u>does</u> rule out the following: Angie is her own parent, her own grandparent (by blood not marriage), etc. We need ‘P’ axioms to take care of these.

Parenthood is notoriously difficult to axiomatize (impossible in my opinion). Without any axioms, you have the following extremely simple problem: P(a,a) might be true. To rule that out, we could add $\forall x\ \neg P(x,x)$. That is, nobody is their own parent. But then it is still consistent that you are the parent of your parent. We want to rule that out so we could add $\forall x \forall y(P(x,y) \rightarrow \neg P(y,x))$. But actually, you don't need both.

5) Show that $\forall x \forall y(P(x,y) \rightarrow \neg P(y,x))$ [lets call this A1] implies that no one can be their own parent $\forall x\ \neg P(x,x)$

6) Show that A1 does rule out being your own grandparent. That is, give a proof that $\forall x \forall y(P(x,y) \rightarrow \neg P(y,x))$ + G entail $\neg \exists x\ G(x,x)$.

7) This single axiom is not complete. Give a model of A1 interpreting P(x,y) as an arrow pointing from x to y which is consistent with someone being the parent of their own grandparent. Obviously to show this, you need to use the definition G. So give your diagram with just the P arrows and name the objects and then also list who are the relevant objects and relationships that make someone the parent of their own grandparent.

8) We want to block this model as well. So lets just explicitly add the axiom that no one is a parent of their grandparent $\neg \exists x \exists y(P(x,y) \wedge G(y,x))$. Call this A2. First, give a proof that this is equivalent to adding $\forall x \forall y \forall z[(P(x,y) \wedge P(y,z)) \rightarrow \neg P(z,x))]$. To do this, show that G+A2 implies $\forall x \forall y \forall z[(P(x,y) \wedge P(y,z)) \rightarrow \neg P(z,x))]$ and also that G+$\forall x \forall y \forall z[(P(x,y) \wedge P(y,z)) \rightarrow \neg P(z,x))]$ entails $\neg \exists x \exists y(P(x,y) \wedge G(y,x))$.

9) But this is not enough either. Give a model of A1+A2 which shows that it is possible that you can be the grandparent of your own grandparent (give an interpretation as in #6)

Everything below here is new

If you think about the examples in 5-9 above, it is clear that to axiomatize parenthood, we need to say that you can't be your own parent, you can't be your own grandparent, you can't be your own great grandparent, ad infinitum. We can do a lot of this by adding an ancestor relationship.

Add the postulate:

Anc: $\forall x \forall y[A(x,y) \leftrightarrow (P(x,y) \vee \exists z(P(x,z) \wedge A(z,y)))]$ *ancestor (purported definition)*

Now add the axiom **Irr**: $\forall x\ \neg A(x,x)$

Show that **Anc+Irr** implies:

*1) $\forall x\ \neg P(x,x)$

*2) $\forall x \forall y(P(x,y) \rightarrow \neg P(y,x))$

**3) $\forall x \forall y \forall z[(P(x,y) \wedge P(y,z)) \rightarrow \neg P(z,x)]$

Give models of **S-C+Anc+Irr that shows that each of the following is possible if we take S-C as stipulative definitions. For each model, name each of the points and give the relevant people that satisfy the relevant definitions. For example, to show that Bob's uncle can be the parent of Bob's grandfather, say who the uncle is and why they are Bob's uncle and who the grandfather is and why that person is Bob's grandfather.

1) Adam is his own uncle
2) Bob's uncle is the parent of Bob's grandfather
3) Christine has a sibling who is also her cousin and also her grandparent

Now these axioms would be pretty good if we wanted to allow models where people had no parents or only one parent. But if we want to be realistic, it seems like we should change that.

Write a sentence that says everyone has at least one parent.
*Write a sentence that says everyone has exactly one parent.
**Write a sentence that says everyone has exactly two parents.

Add the 'exactly one parents' axiom. Give a model of **Anc+Irr+exactly one.
[hard] Take {Anc+Irr+exactly two**} to be the purported axioms. I believe that this is not a good axiomatization of Parent+Ancestor.

1) Explain why one of these is false on the domain of currently living humans.
2) Is it plausible that these are true on any domain where P means Parent and A means Ancestor?
3) Assume that infinite domains and infinite chains of parents are allowed. Still, this is not a good axiomatization. We should be thinking about two parents per child, but for simplicity, think of the one parent model. The same problem arises, namely, **Anc** is false. Give a model of the axioms where the A relation holds when it really shouldn't.